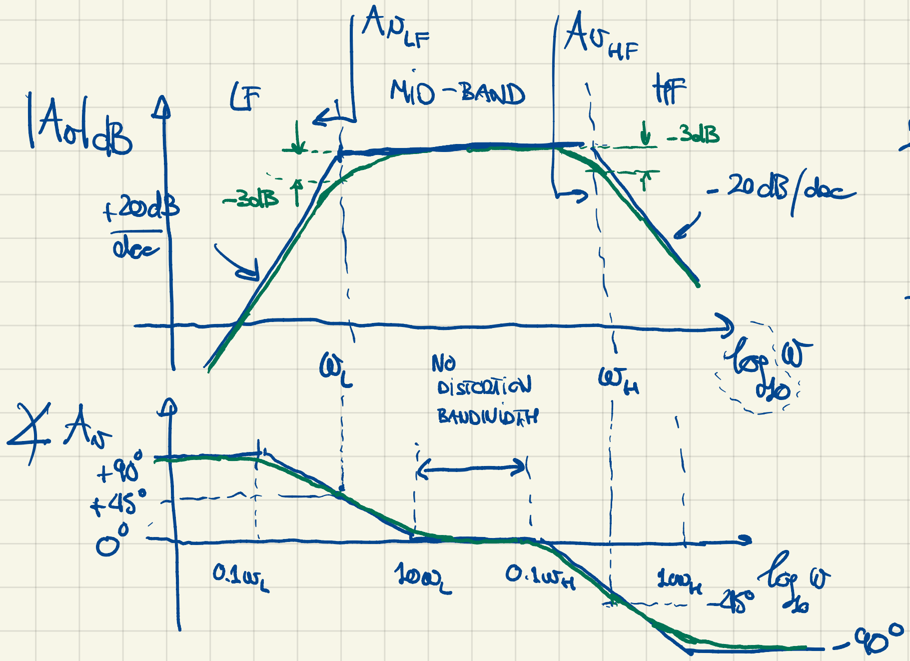
SEPTEMBER 30th, 2022

LESSON #1

- COURSE ORGANIZATION
- QUESTION TIME → EVERY 2 MONDAYS AT 9:30 ON ZOOM

• FREQUENCY RESPONSE RECAP



$$20\log_{10}|A_v(j\omega)|$$

$$BW \triangleq \omega_H - \omega_L \cong \omega_H$$

MINIMUM COMPLEXITY TRANSFER FUNCTION

$$A_v(s) = A_{v_{MB}} \cdot \frac{s\tau_L}{(1+s\tau_L)(1+s\tau_H)} = \underbrace{A_{v_{MB}} \cdot \frac{s\tau_L}{1+s\tau_L}}_{A_{v_{LF}}(s)} \cdot \underbrace{\frac{1}{1+s\tau_H}}_{A_{v_{HF}}(s)}$$

$$\tau_L = \frac{1}{\omega_L} \qquad \tau_H = \frac{1}{\omega_H}$$

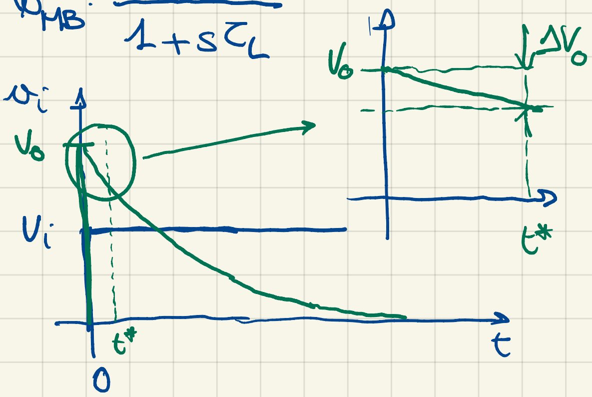
◊ LOW FREQUENCY RESPONSE → $A_{v_{LF}}(s) = A_{v_{MB}} \cdot \frac{s\tau_L}{1+s\tau_L}$

STEP RESPONSE ANALYSIS

$$\theta(s) = \mathcal{L}\left[v_i(t)\right] = \frac{V_i}{s}$$

$$V_o(s) = \theta(s) \cdot A_{v_{LF}}(s) = A_{v_{MB}} \cdot \frac{\tau_L}{1+s\tau_L} \cdot V_i$$

$$\mathcal{L}^{-1}\left[V_o(s)\right] = \underbrace{V_i\, A_{v_{MB}}}_{V_o} \cdot e^{-\frac{t}{\tau_L}}$$

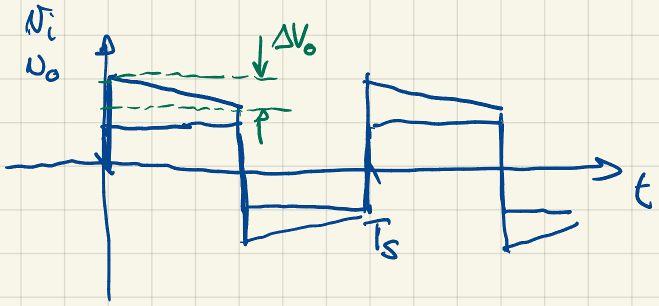


$$\boxed{t^* \ll \tau_L}$$

$$v_o(t) = V_o \cdot \Big(1 - \frac{t}{\tau_L} + \underbrace{\cdots\cdots}_{\text{NEGLIGIBLE}}\Big) \cong V_o\Big(1 - \frac{t}{\tau_L}\Big) \quad \forall\ t \in [0, t^*]$$

$$\Delta V_o = V_o - v_o(t^*) = \cancel{V_o} - \cancel{V_o} + V_o \frac{t^*}{\tau_L}$$

$$\frac{\Delta V_o}{V_o} = \frac{t^*}{\tau_L} \implies \boxed{\omega_L = \frac{1}{t^*} \cdot \frac{\Delta V_o}{V_o}}$$



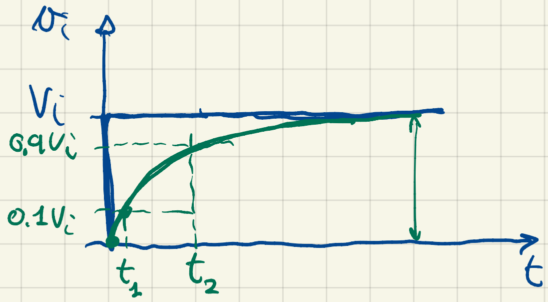
$$T_s \ll \tau_L$$

$$t^* = \frac{T_s}{2} \implies \boxed{\omega_L = \frac{2}{T_s} \cdot \frac{\Delta V_o}{V_o}}$$

◊ HIGH FREQUENCY RESPONSE

$$A_{v_{HF}}(s) = \frac{1}{1 + s\tau_H}$$

STEP RESPONSE ANALYSIS



$$\theta(s) = \frac{V_i}{s}$$

$$V_o(s) = \frac{V_i}{s(1 + s\tau_H)} = V_i \left[\frac{1}{s} - \frac{\tau_H}{1 + s\tau_H}\right]$$

$$\mathcal{L}^{-1}\left[V_o(s)\right] = V_i\left(1 - e^{-\frac{t}{\tau_H}}\right)$$

$$t_r \triangleq t_2 - t_1$$ RISE TIME OF THE STEP RESPONSE

$$\begin{cases} v_o(t_1) = V_i\left(1 - e^{-\frac{t_1}{\tau_H}}\right) = 0.1\,V_i \\ v_o(t_2) = V_i\left(1 - e^{-\frac{t_2}{\tau_H}}\right) = 0.9\,V_i \end{cases}$$

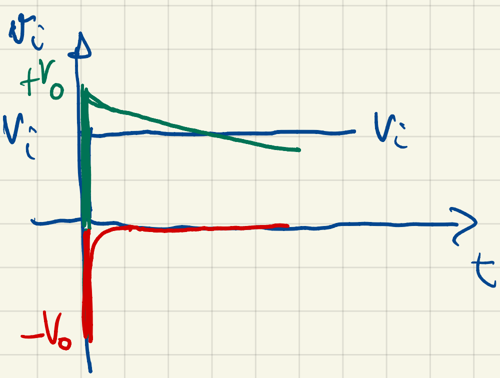
$$\begin{cases} 0.9\,V_i = e^{-\frac{t_1}{\tau_H}} \\ 0.1\,V_i = e^{-\frac{t_2}{\tau_H}} \end{cases} \Rightarrow \quad 9 = e^{\frac{t_2 - t_1}{\tau_H}} \simeq e^{\frac{t_2}{\tau_H}}$$

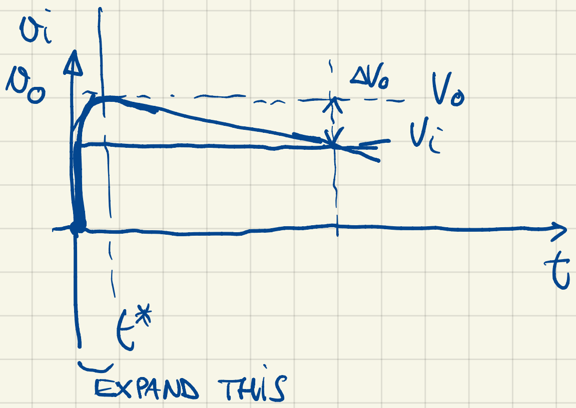
$$\underset{\substack{\wr\wr \\ 2.2}}{\ln(9)} = \frac{t_2}{\tau_H} \quad \Longrightarrow \quad \omega_H = \frac{\ln(9)}{t_2} \simeq \frac{2.2}{t_2}$$

WE CAN PUT IT ALL TOGETHER AND FIND

$$V_o(s) = \frac{V_i}{s} A_v(s) = V_i \frac{\tau_L}{(1+s\tau_L)(1+s\tau_H)} \cdot A_{v_{MB}}$$

$$\mathcal{L}^{-1}\left[V_o(s)\right] = \underbrace{A_{v_{MB}} V_i}_{V_o} \underset{\substack{\wr\wr \\ 1}}{\frac{\tau_L}{\tau_L - \tau_H}} \left(\underline{e^{-\frac{t}{\tau_L}}} - \underline{e^{-\frac{t}{\tau_H}}} \right)$$



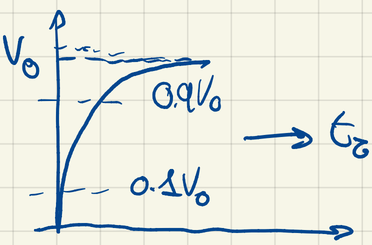


$$\frac{\Delta V_o}{V_o} \rightarrow \omega_L$$

IN CONCLUSION:

BECAUSE $\tau_L >> \tau_H$, FROM

A SINGLE STEP RESPONSE



$$\rightarrow t_2 \rightarrow \omega_H$$

WE CAN EXTRACT VERY ACCURATE ESTIMATION OF BOTH ω_L AND ω_H

DISTORTION ANALYSIS

Hyp: THE AMPLIFIER IS OPERATING AS A LTI SYSTEM

$$v_i(t) = \sum_{k=1}^{+\infty} V_{ik} \sin\left(2\pi k f_0 t + \varphi_{ik}\right)$$

T_0 is THE PERIOD OF SIGNAL v_i → f_0 IS ITS FUNDAMENTAL FREQUENCY

APPLYING SUPERPOSITION (AMPLIFIER IS LINEAR TIME INVARIANT)

$$v_o(t) = \sum_{k=1}^{+\infty} \underbrace{|A_v(j2\pi k f_0)|\, V_{ik}}_{V_{ok}} \sin\left(2\pi k f_0 t + \varphi_{ik} + \Phi_{Ak}\right)$$

IN ORDER TO HAVE NO DISTORTION

$$\begin{cases} v_o(t) = A\, v_i(t) \\ v_o(t) = A\, v_i(t - t_A) \end{cases}$$

OR ELSE

CONSTANT

TIME SHIFT (AKA LINEAR PHASE)

WHICH MEANS

$$\begin{cases} 1. \quad |A_v(j2\pi k f_0)| = \text{CONSTANT } \forall k \\ 2. \quad \dfrac{\phi_{Ak}}{2\pi k f_0} = \text{CONSTANT } \forall k \end{cases}$$

⇒ LINEAR PHASE RESPONSE

$$\frac{\phi_{Ak}}{2\pi k f_0} = -t_A \longrightarrow \phi_{Ak} = -2\pi k f_0 t_A$$

$$\longrightarrow \sin\left(2\pi k f_0 (t - t_A) + \varphi_{ik}\right)$$

IN PARTICULAR, $\phi_{Ak} = n \cdot 2\pi \qquad n \in [0, 1, 2, 3 \ldots]$

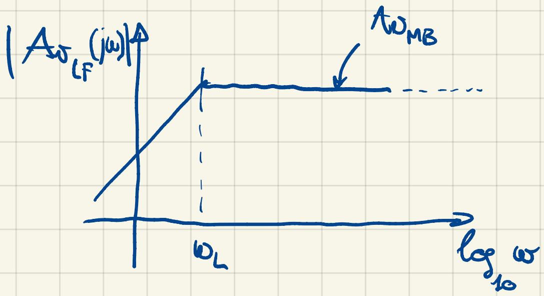
GUARANTEES NO DISTORTION.

◊ SHORT CIRCUIT TIME CONSTANT (SCTC) METHOD

PURPOSE: DERIVING AN ESTIMATION OF ω_L FROM THE CIRCUIT SCHEMATIC

$$A_{v_{LF}}(s) = A_{v_{MB}} \frac{N(s)}{1 + a_1 s + a_2 s^2 + \dots + a_n s^n}$$

LOW FREQUENCY RESPONSE



ORDER OF DENOMINATOR IS n

ORDER OF NUMERATOR IS m

$$m = n$$

$$\lim_{s \to +\infty} A_{v_{LF}}(s) = \text{CONST}$$

COEFFICIENTS a_i ARE REAL NUMBERS $\Rightarrow$ POLES ARE COMPLEX IN GENERAL, BUT ALWAYS IN COUPLES (CONJUGATE POLES)

IN AMPLIFIERS WHERE NO FEEDBACK IS APPLIED, POLES ARE OFTEN PURELY REAL AND, IN ANY CASE, NEGATIVE REAL PART. INDEED ANY ELECTRONIC AMPLIFIER IS NORMALLY OPEN LOOP STABLE