

// x_i : #persone turno $i \in \{1..6\}$ ($1=2-6, 2=6-10 \dots$)

ES. 1

// $y_i = 1$ se turno $i \in \{1..6\}$ è usato, 0 altrimenti

// $z_i = 1$ se turno $i \in \{1..6\}$ ha almeno 11 persone, 0 altrimenti

$$\min x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + 0,1(y_1 + y_2 + y_3 + y_4 + y_5 + y_6) + 0,1(x_1 + x_4) + 0,2(x_5 + x_6)$$

$$\text{s.t. } x_6 + x_1 \geq 5; x_1 + x_2 \geq 15; x_2 + x_3 \geq 12; x_3 + x_4 \geq 19; x_4 + x_5 \geq 10; x_5 + x_6 \geq 7$$

$$x_1 \leq 12$$

$$x_6 \leq 12$$

$$x_i \leq M y_i, i=1..6 \quad \# M \text{ costante suff. grande (e.g. } M=19)$$

$$\begin{aligned} & y_1 + y_2 + y_3 + y_4 + y_5 + y_6 \leq 4 \\ & z_1 + z_2 + z_3 + z_4 + z_5 + z_6 \geq 2 \\ & x_i \in \mathbb{Z}_+, y_i \in \{0,1\}, z_i \in \{0,1\} \end{aligned}$$

$$\text{F.S.: } \min \hat{x}_1 - x_2 - 3x_3$$

$$\text{s.t. } -2\hat{x}_1 - 3x_2 - 3x_3 + x_4 = 2$$

$$x_2 + x_3 + x_5 = 2$$

$$\hat{x}_1 + x_2 + 2x_3 + x_6 = 4$$

	\hat{x}_1	x_2	x_3	x_4	x_5	x_6	z	\bar{b}	
	1	-1	-3	0	0	0	-1	0	
x_4	-2	-3	-3	1	0	0	0	2	
x_5	0	1	1	0	1	0	0	2	
x_6	1	1	2	0	0	1	0	4	
	1	0	-2	0	1	0	-1	2	$+R_2$
x_4	-2	0	0	1	3	0	0	8	$+3R_2$
x_2	0	1	1	0	1	0	0	2	R_2
x_6	1	0	1	0	-1	1	0	2	$-R_2$
	1	2	0	0	3	0	-1	6	$+2R_2$
x_4	-2	0	0	1	3	0	0	8	R_1
x_3	0	1	1	0	1	0	0	2	R_2
x_6	1	-1	0	0	-2	1	0	0	$-R_2$

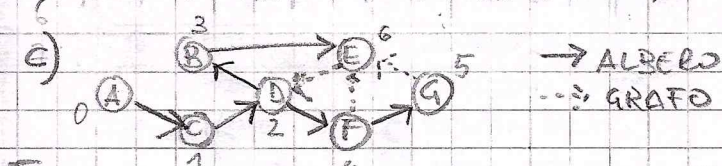
$$a) x_1 = -\hat{x}_1 = 0, x_2 = x_5 = 0, x_3 = 2, x_4 = 8, x_6 = 0, z = -6$$

$$b) w^* = z^* = -6 \text{ (DUALITA' FORTE)}$$

$$\text{ES.3 } a) \text{ Bellman-Ford } (\exists a_{ij} < 0)$$

b) ↓

it	A	B	C	D	E	F	G	Agg.
0	0A	∞	∞	∞	∞	∞	∞	A
1	0A	5A	1A	4A	∞	∞	∞	BCD
2	0A	5A	1A	2C	8B	5C	∞	DEF
3	0A	3D	1A	2C	7F	4D	6F	BEFG
4	0A	3D	1A	2C	6B	4D	5F	EG
5	0A	3D	1A	2C	6B	4D	5F	✓



$$d) G \leftarrow F \leftarrow C \leftarrow A : A \leftarrow C \leftarrow F, \text{ costo } 6$$

$$\text{ES.6 } a) \text{ UB} \downarrow, \text{LB(S.A.)} \uparrow \text{ (NE[2,6,2])}$$

$$b) \text{ ottimo } \in [2,6]$$

$$c) P_3, P_4, P_5, P_6; \text{ chiuso } P_3 = P_4 \text{ (N.M.)}$$

$$d) P_5$$

$$e) \text{UB}_5 = \text{LB}_5 \in [2,6]$$

CONCLUSIONI ES.6 7CFU:

$$\text{OTTIMO} = 21 (\text{LB}_0). b) \text{ radice } \in [2,23]; \text{maxi } \in [2,22]$$

$$\text{ES.4 } a) \text{ enunciato ...}$$

$$b) \cdot 0, -2, 0 \dots \text{ AMMISSIBILE } \checkmark$$

$$\begin{aligned} & \text{duale: } \min 2u_1 + u_2 + 3u_3 - 2u_4 \\ & \text{s.t. } 2u_2 + u_3 + u_4 = 2 \\ & -u_1 - 2u_2 + u_4 \leq 1 \\ & -u_1 + 2u_2 + 3u_3 \geq -1 \\ & u_1 \geq 0, u_2 \leq 0, u_3 \geq 0, u_4 \text{ libera} \end{aligned}$$

$$\cdot \text{CCPD: } x_1 \text{ libera (no CCPD)}$$

$$x_2 < 0 \Rightarrow -u_1 - 2u_2 + u_4 = 1$$

$$x_3 = 0 \Rightarrow //$$

$$\cdot (-x_2 - x_3 - 2)u_1 = 0 \Rightarrow 0u_1 = 0 \Rightarrow //$$

$$\cdot (2x_4 - 2x_2 - 1)u_2 = 0 \Rightarrow 3u_2 = 0 \Rightarrow u_2 = 0$$

$$\cdot (x_1 + 2x_2 - 3)u_3 = 0 \Rightarrow -3u_3 = 0 \Rightarrow u_3 = 0$$

$$\cdot \text{vincolo} = \text{no CCPD}$$

$$\begin{aligned} & \text{Sistema: } \begin{cases} -u_1 - 2u_2 + u_4 = 1 & (\text{CCPD}) \\ u_2 = 0, u_3 = 0 & (\text{CCPD}) \\ 2u_2 + u_3 + u_4 = 2 & (\text{A.O.}) \end{cases} \\ & \Rightarrow u_4 = 2, u_1 = 1, u_2 = u_3 = 0 \end{aligned}$$

$$b) \text{ AMM. DUALI: } u_1 \geq 0, u_2 \leq 0, u_3 \geq 0, u_4 \checkmark$$

$$\text{vincoli OK } (1, -1, 0, 2 \geq -1)$$

$$\cdot \text{WNC: OTTIMO!}$$

$$\text{ES.5 } a) \text{ F.CAN. con } B = [x_6, x_3, x_2],$$

$$\exists \bar{c} < 0 \text{ e } \exists \theta < 0 \Rightarrow \text{NON OTT.}$$

$$b) \text{ porta a base N.A. ...}$$

$$c) x_1, x_3; x_1, x_2; x_4, x_6$$

$$d) x_1, x_2; z = -6 - 10 \cdot \frac{1}{10} = -7$$

$$\text{7CFU } [x_3 = 0 \text{ in base}]$$

$$\text{ES.6 (7CFU) } a) \text{ ORDINE } x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow x_1$$

$$P_0: x_2 = 1 \quad \bar{w} = 12 \quad \text{UB}_0 = 17 + \frac{1}{4} \cdot 7 = 23$$

$$x_3 = 1 \quad \bar{w} = 6 \quad \text{LB}_0 = 17 + 4 = 21$$

$$x_4 = \frac{6}{7}$$

$$P_1: x_2 = x_3 = 1 \quad \bar{w} = 6$$

$$x_5 = \frac{5}{7}$$

$$\text{UB}_1 = 17 + \frac{6}{7} \cdot 6 = 22$$

$$\text{LB}_1 = 17 + 4 = 21$$

$$P_2: x_4 = 1 \rightarrow \bar{w} = 10$$

$$x_2 = 1 \quad \bar{w} = 5$$

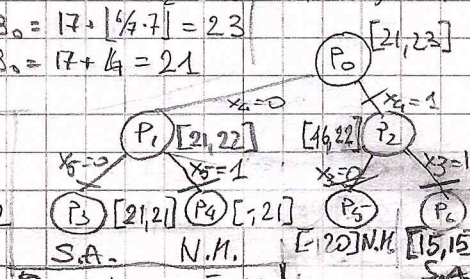
$$x_3 = \frac{5}{6}$$

$$\text{UB}_2 = 7 + 9 + \frac{5}{6} \cdot 8 = 21$$

$$P_3: x_4 = x_5 = 0$$

$$x_2 = x_3 = 1 \rightarrow \bar{w} = 6$$

$$x_6 = 1 \quad \text{UB}_3 = \text{LB}_3 = 21$$



$$P_7: x_4 = x_5 = 0 \quad \bar{w} = 10$$

$$x_2 = 1 \quad \bar{w} = 5$$

$$x_3 = \frac{5}{7}$$

$$\text{UB}_5 = 7 + 9 + \frac{5}{7} \cdot 6 = 20 \Rightarrow \text{UB} = \text{LB} = 15$$

$$P_4: x_4 = 0, x_5 = 1 \quad \bar{w} = 10$$

$$x_2 = 1, \bar{w} = 5$$

$$x_3 = \frac{5}{6}$$

$$\text{UB}_4 = 6 + 9 + \frac{5}{6} \cdot 8 = 21$$