Network Science

#8 Other Centrality Measures

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Eigenvector and Katz centralities

Eigenvector and Katz centralities

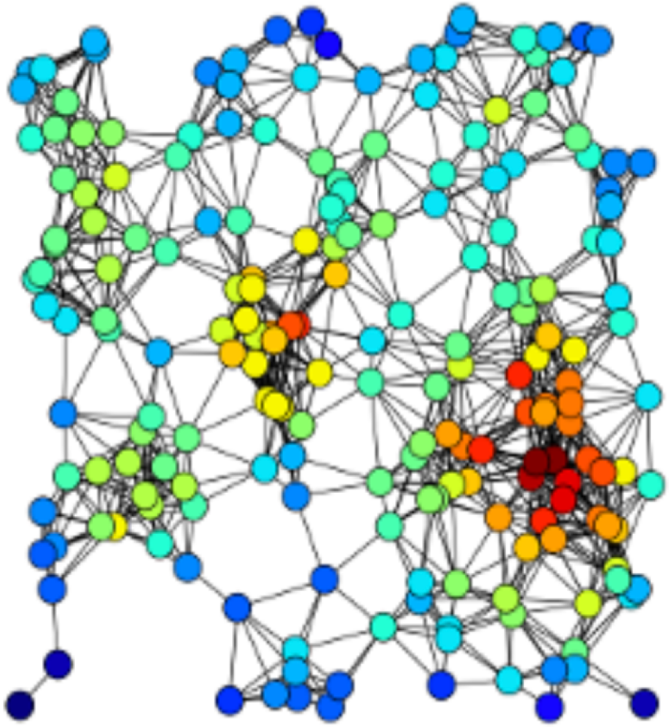
	with constant term	without constant term
normalized	PageRank $\mathbf{r} = c\,\mathbf{M}\,\mathbf{r} + (1-c)\,\mathbf{q}$	Degree $\mathbf{r} = \mathbf{M}\,\mathbf{r}$
unnormalized	Katz $\mathbf{r} = c\,\mathbf{A}\,\mathbf{r} + \mathbf{1}$	Eigenvector $\mathbf{r} = c\,\mathbf{A}\,\mathbf{r}$

$$\mathbf{r} = (\mathbf{I} - c\,\mathbf{A})^{-1}\,\mathbf{1}$$
$$= \sum (c\,\mathbf{A})^{k}\,\mathbf{1}$$

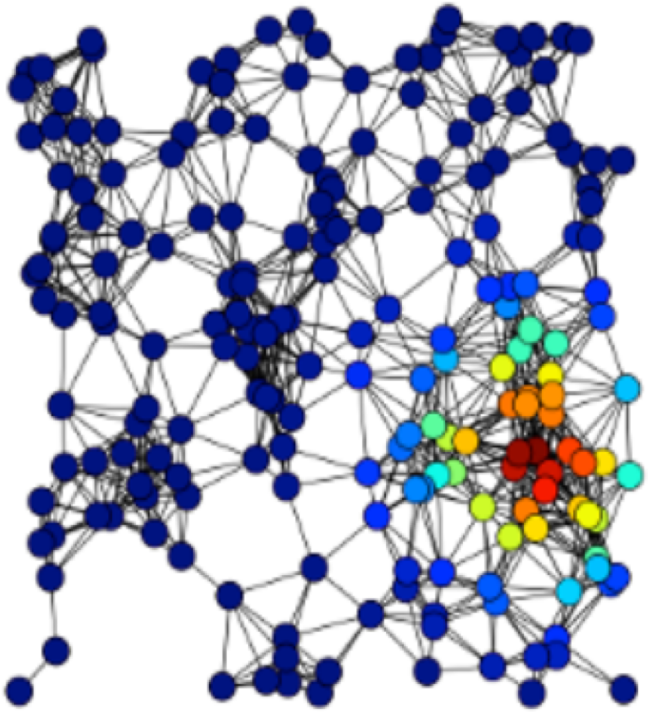
MiME.

Comparison

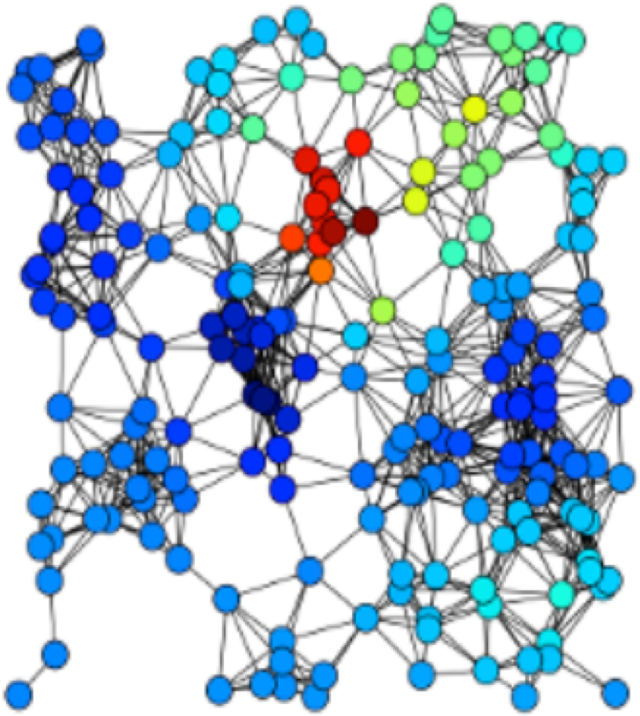
Degree



Eigenvector



Katz



Closeness centrality

What is Closeness?

Closeness centrality

From Wikipedia, the free encyclopedia

In a connected graph, **closeness centrality** (or **closeness**) of a node is a measure of centrality in a network, calculated as the reciprocal of the sum of the length of the shortest paths between the node and all other nodes in the graph. Thus, the more central a node is, the *closer* it is to all other nodes.

Closeness was defined by Bavelas (1950) as the reciprocal of the **farness**,[1][2] that is:

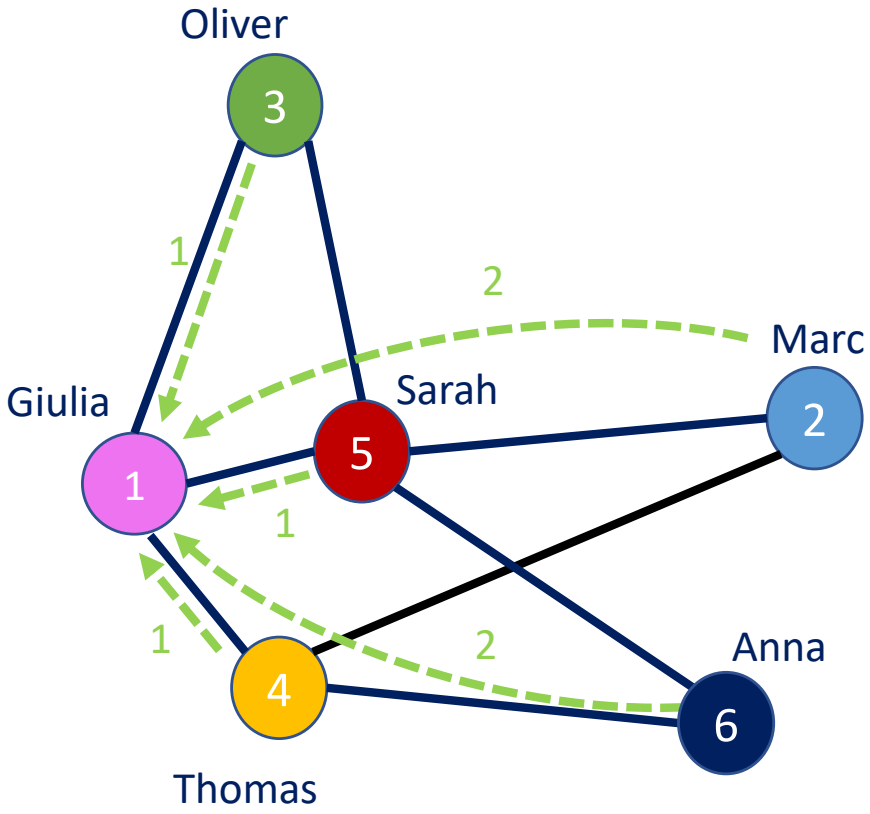
$$C(x) = \frac{1}{\sum_y d(y,x)}.$$

where $d(y,x)$ is the distance between vertices x and y.

Rationale: the node which is the easiest to reach, the one which is the best for spreading information

Example

count the lengths of the shortest paths leading to Giulia
1 + 2 + 1 + 2 + 1 = 7



Closeness

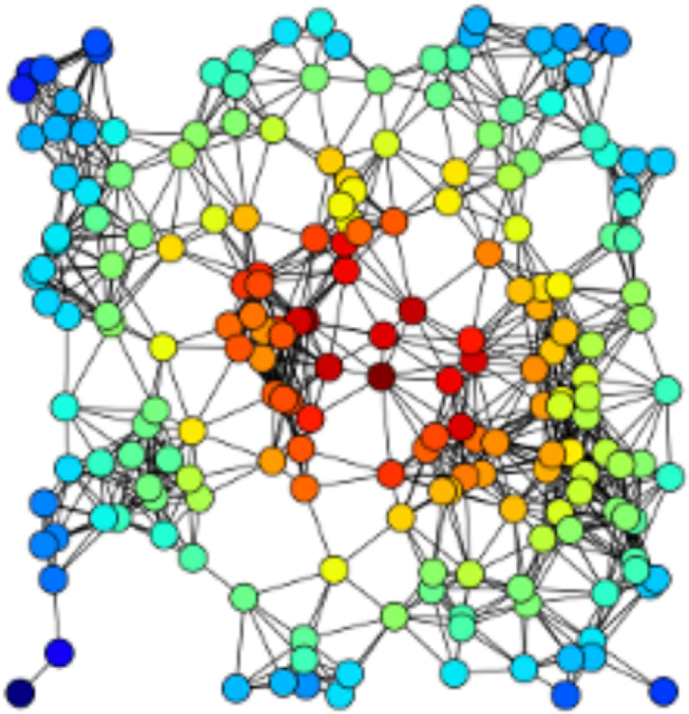
0.1429	Giulia
0.1250	Marc
0.1250	Oliver
0.1429	Thomas
0.1667	Sarah
0.1250	Anna

Sarah is the preferred node for spreading information

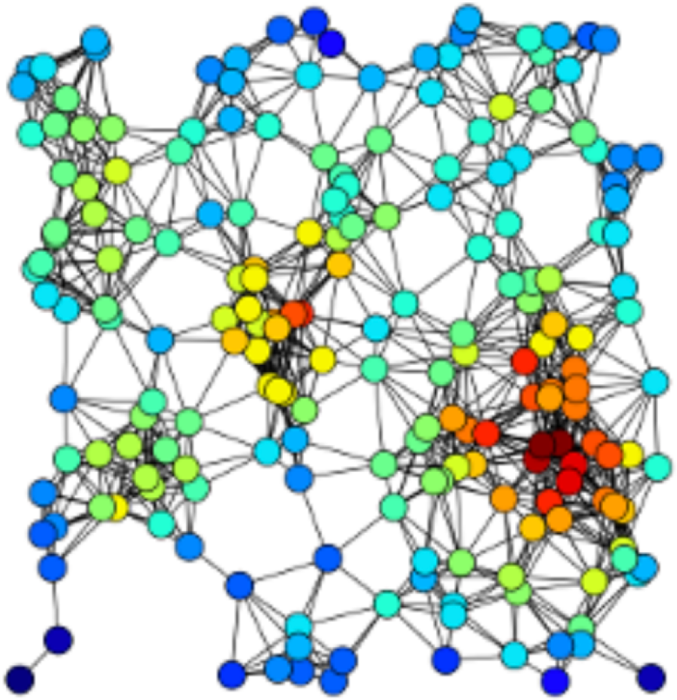
C(Giulia) = 1/7
= 0.1429

Closeness versus Degree centrality

Closeness



Degree



Harmonic centrality

In disconnected graphs [edit]

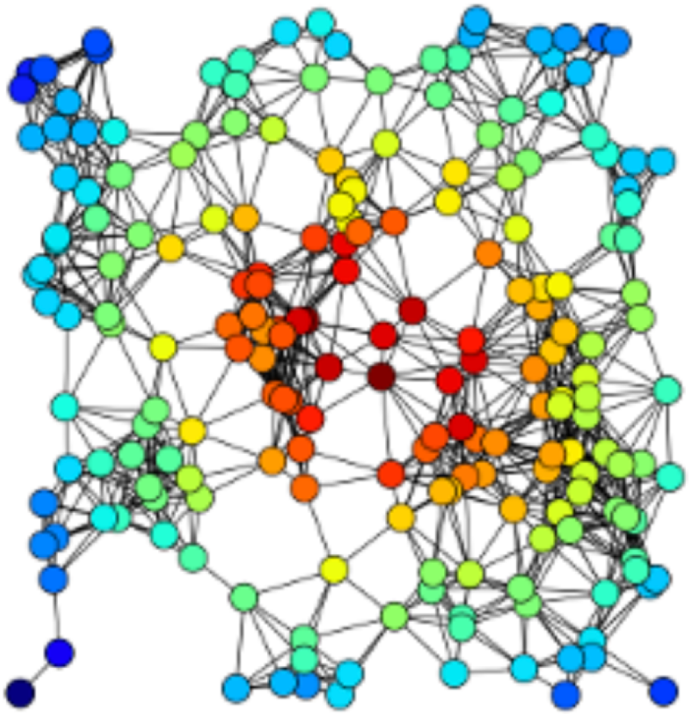
When a graph is not strongly connected, a widespread idea is that of using the sum reciprocal of distances, instead of the reciprocal of the sum of distances, with the convention $1/\infty = 0$:

$$H(x) = \sum_{y \neq x} \frac{1}{d(y,x)}.$$

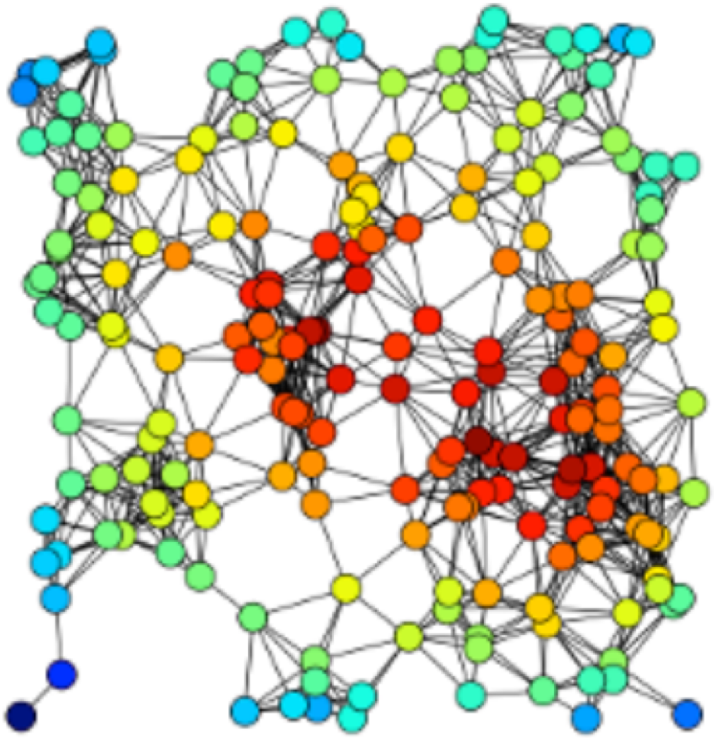
The most natural modification of Bavelas's definition of closeness is following the general principle proposed by Marchiori and Latora (2000)[3] that in graphs with infinite distances the harmonic mean behaves better than the arithmetic mean. Indeed, Bavelas's closeness can be described as the denormalized reciprocal of the arithmetic mean of distances, whereas harmonic centrality is the denormalized reciprocal of the harmonic mean of distances.

Closeness versus Harmonic centrality

Closeness



Harmonic



Betweenness centrality

Freeman, “A set of measures of centrality based on betweenness,” 1977

https://www.jstor.org/stable/pdf/3033543.pdf

What is Betweenness?

Betweenness centrality

From Wikipedia, the free encyclopedia

In graph theory, **betweenness centrality** is a measure of centrality in a graph based on shortest paths. For every pair of vertices in a connected graph, there exists at least one shortest path between the vertices such that either the number of edges that the path passes through (for unweighted graphs) or the sum of the weights of the edges (for weighted graphs) is minimized. The betweenness centrality for each vertex is the number of these shortest paths that pass through the vertex.

Definition [edit]

The betweenness centrality of a node v is given by the expression:

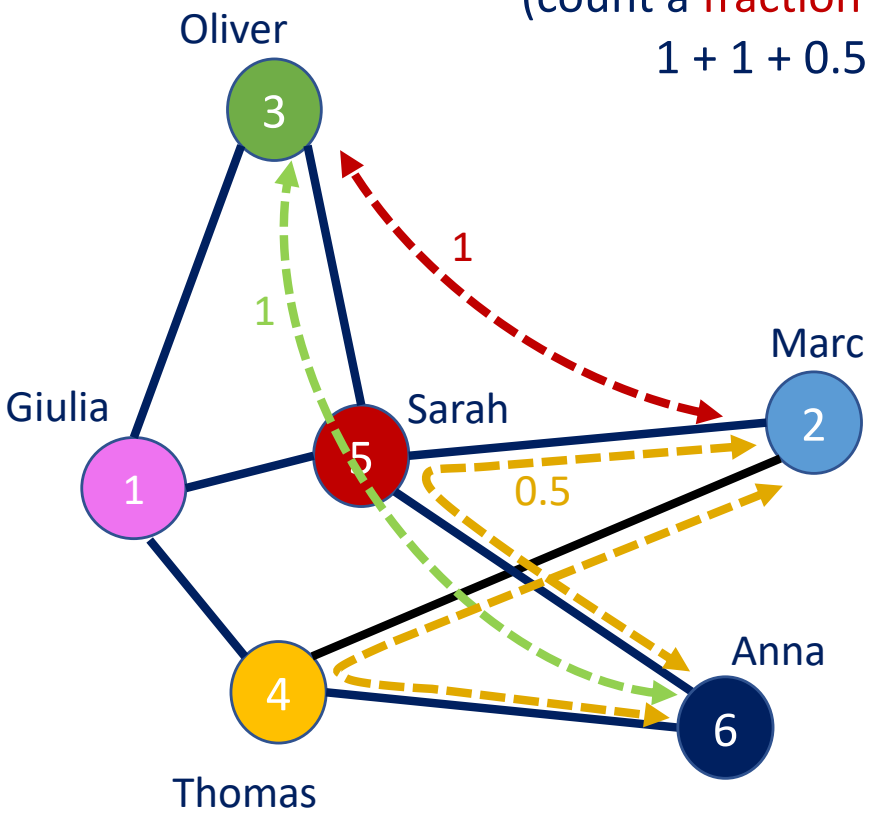
$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ_{st} is the total number of shortest paths from node s to node t and $\sigma_{st}(v)$ is the number of those paths that pass through v.

Rationale: the node which takes you elsewhere (bridge, broker)

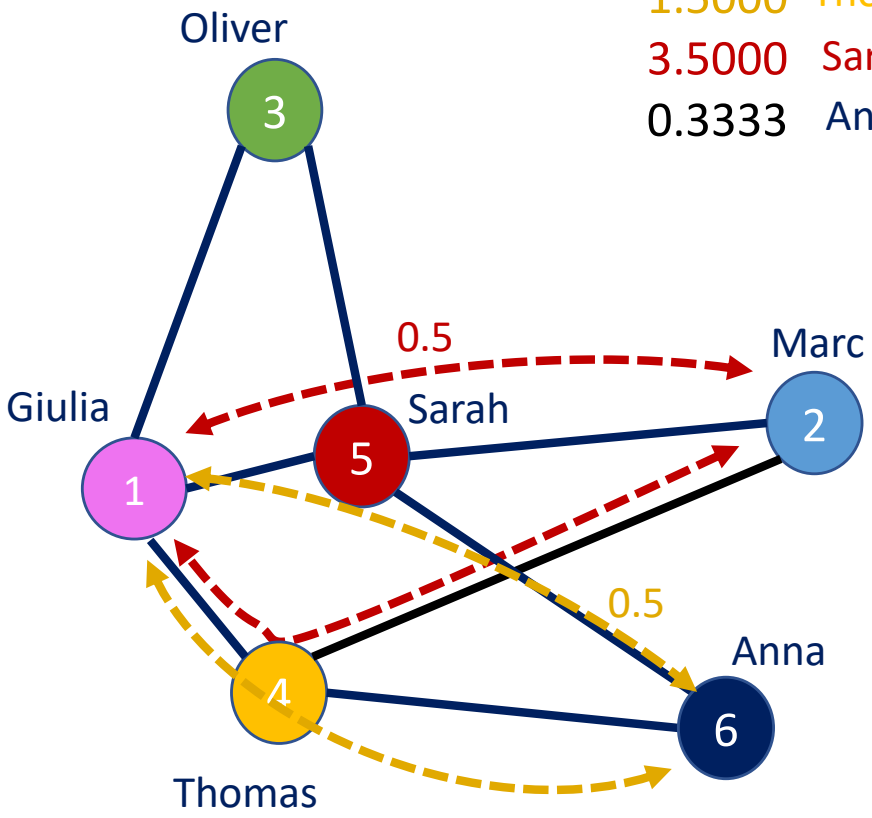
Example

count the # of shortest paths
passing through Sarah
(count a fraction if more than one path)
1 + 1 + 0.5 + 0.5 + 0.5 = 3.5



Betweenness

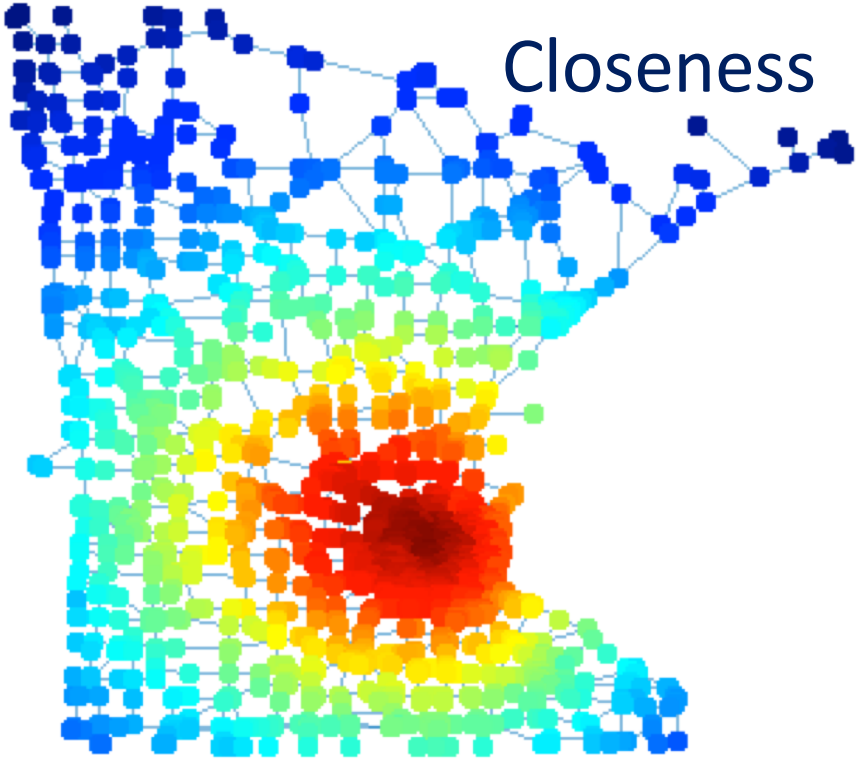
1.3333	Giulia
0.3333	Marc
0	Oliver
1.5000	Thomas
3.5000	Sarah
0.3333	Anna



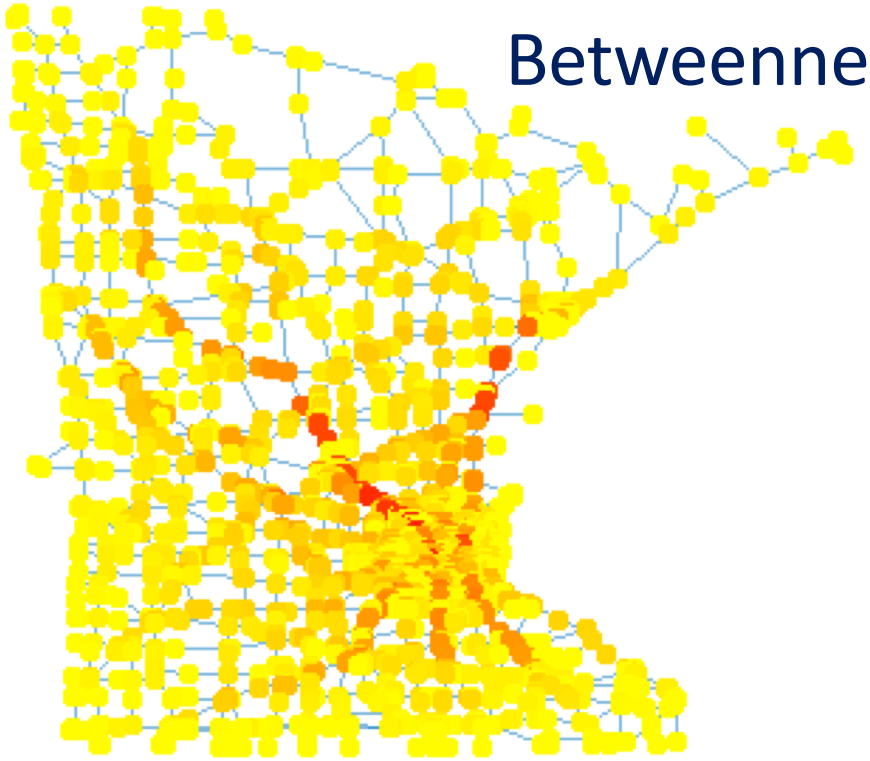
Closeness versus Betweenness centrality

Minnesota road network

Closeness



Betweenness



Closeness is a measure of center of gravity (best node from which to spread info)

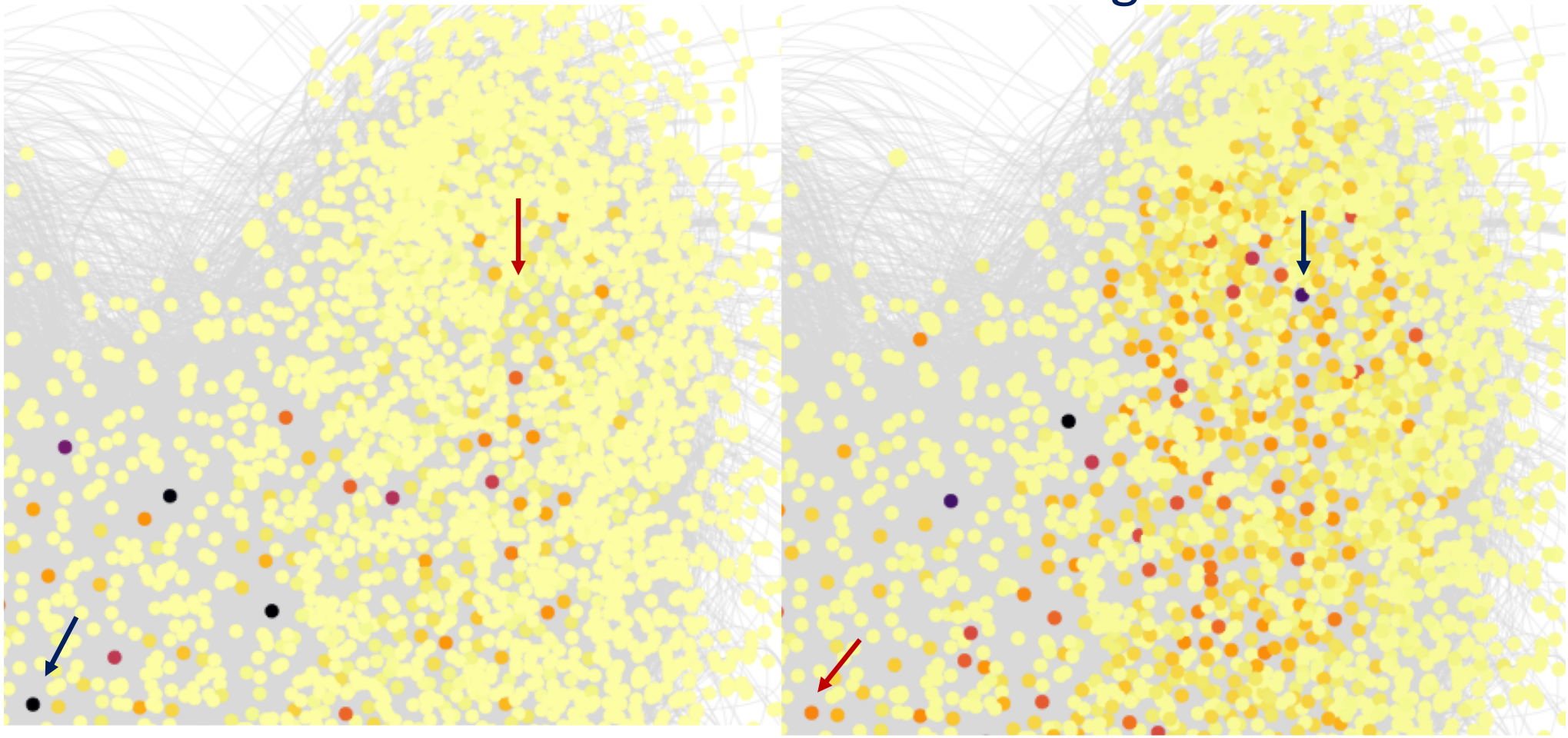
Betweenness is a measure of brokerage (i.e., being a bridge)

MiME.

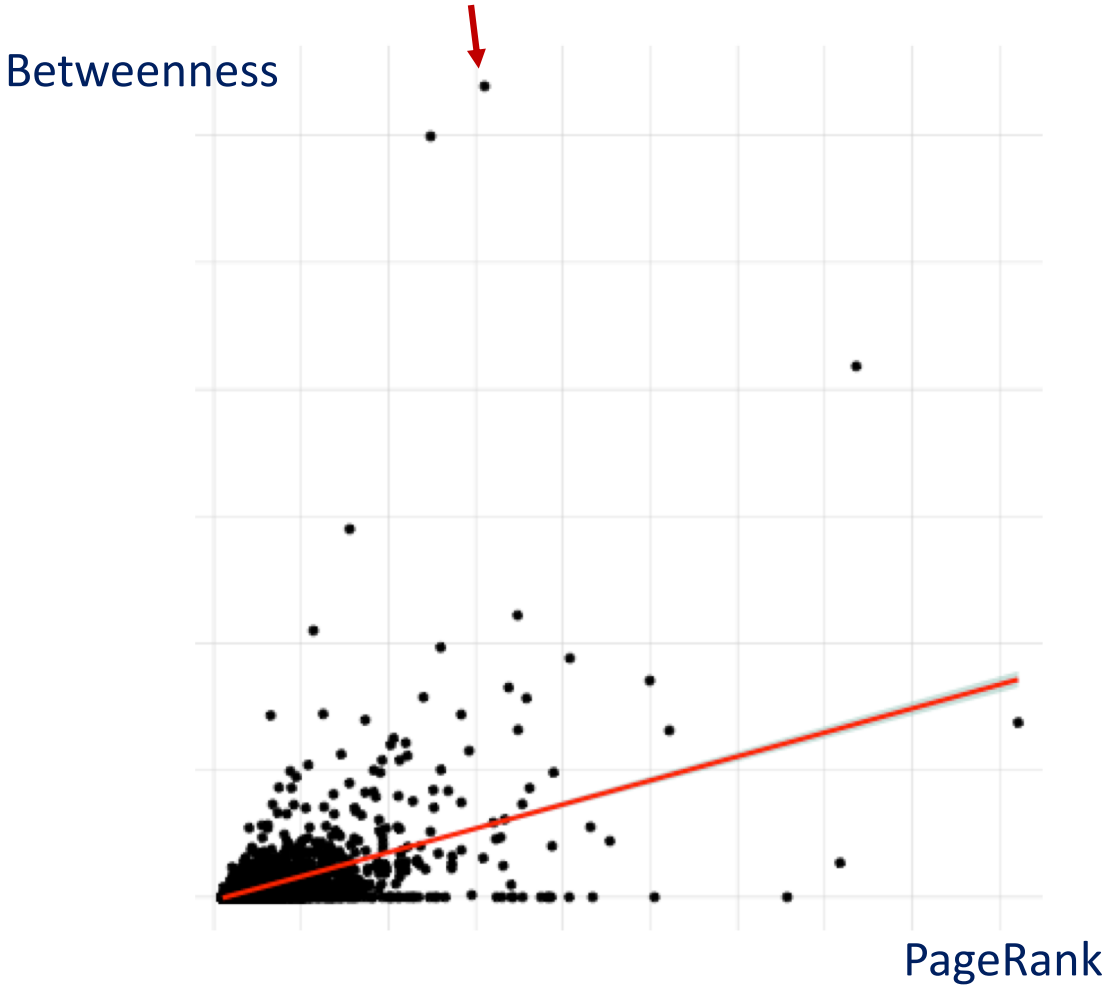
Betweenness versus PageRank centrality

Betweenness

PageRank



Betweenness versus PageRank centrality

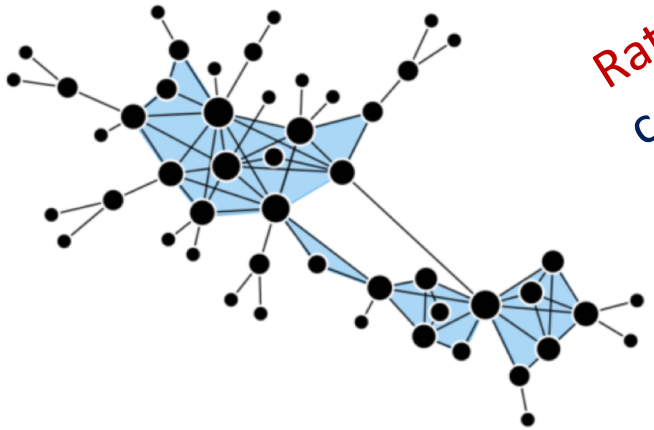


Clustering coefficient

What is the Clustering coefficient?

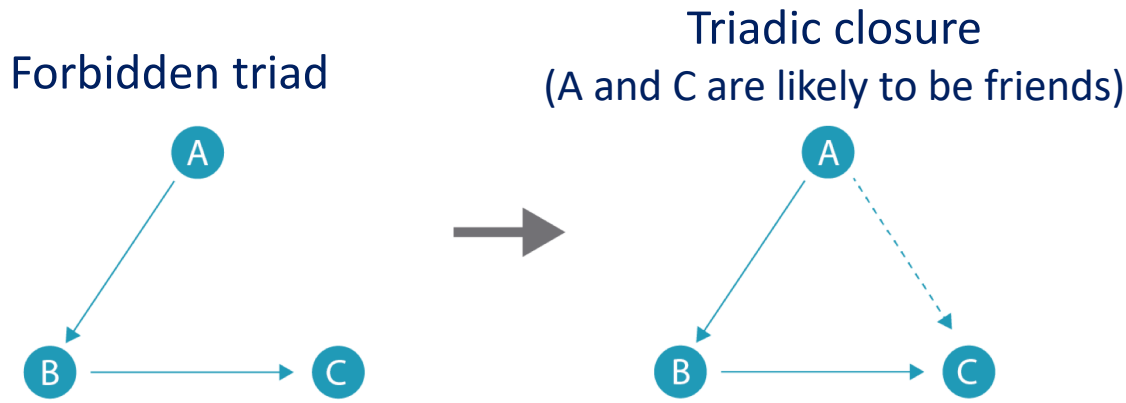
Local clustering coefficient [edit]

The **local clustering coefficient** of a vertex (node) in a graph quantifies how close its neighbours are to being a clique (complete graph). Duncan J. Watts and Steven Strogatz introduced the measure in 1998 to determine whether a graph is a small-world network.



Rationale: how strongly connected is the network locally / general indication of the graph's tendency to be organized into clusters

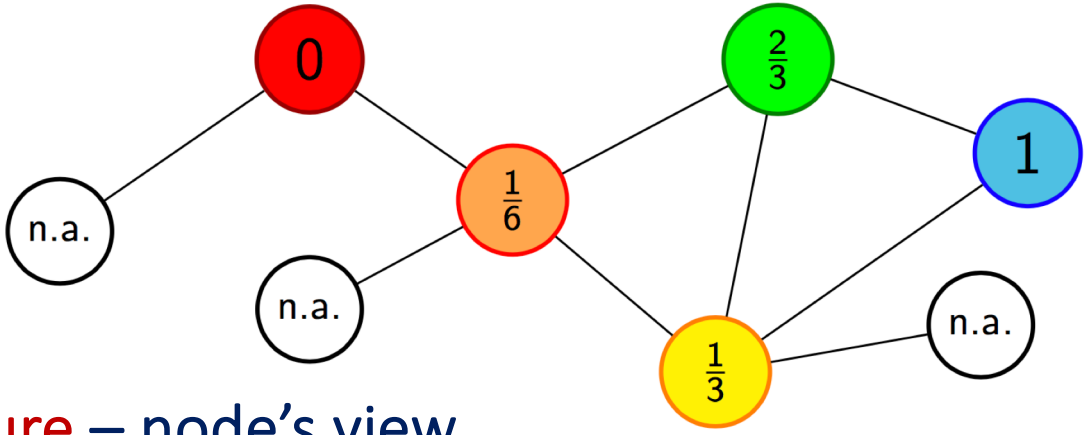
Triadic closure



Triadic closure

- A and C are likely to have the opportunity to meet because they have a common friend B
- The fact that A and C is friends with B gives them the basis of trusting each other
- B may have the incentive to bring A and C together, as it may be hard for B to maintain disjoint relationships

Clustering coefficient and triadic closure



A measure for triadic closure – node's view

- Clustering coefficient C_i
- Counts the fraction of pairs of neighbours which form a triadic closure with node *i*

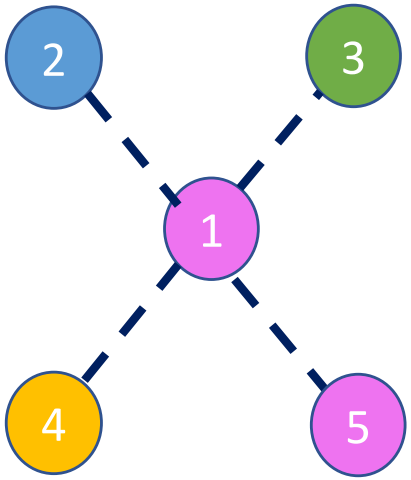
$$C_i = \frac{1}{|\mathcal{N}_i|(|\mathcal{N}_i| - 1)} \sum_{\substack{(j,k) \in \mathcal{N}_i^2 \\ j \neq k}} \mathrm{tc}_{i,j,k}$$

where $tc_{ijk} = 1$ if the triplet (i,j,k) forms a triadic closure, and zero otherwise

Examples

not connected neighbourhood

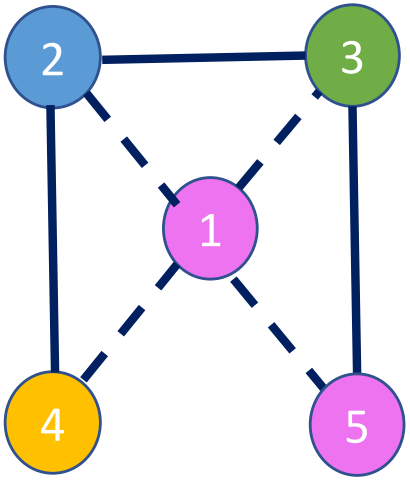
$C_1 = 0$



$\langle C \rangle = 0$

weakly connected neighbourhood

$C_1 = ½ = 3 / (4x3/2)$

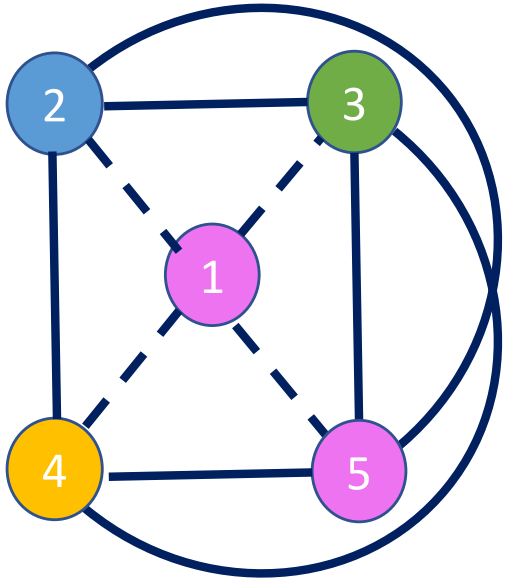


$C_2 = C_3 = ⅔,\ C_4 = C_5 = 1$

$\langle C \rangle = 0.766$

strongly connected neighbourhood

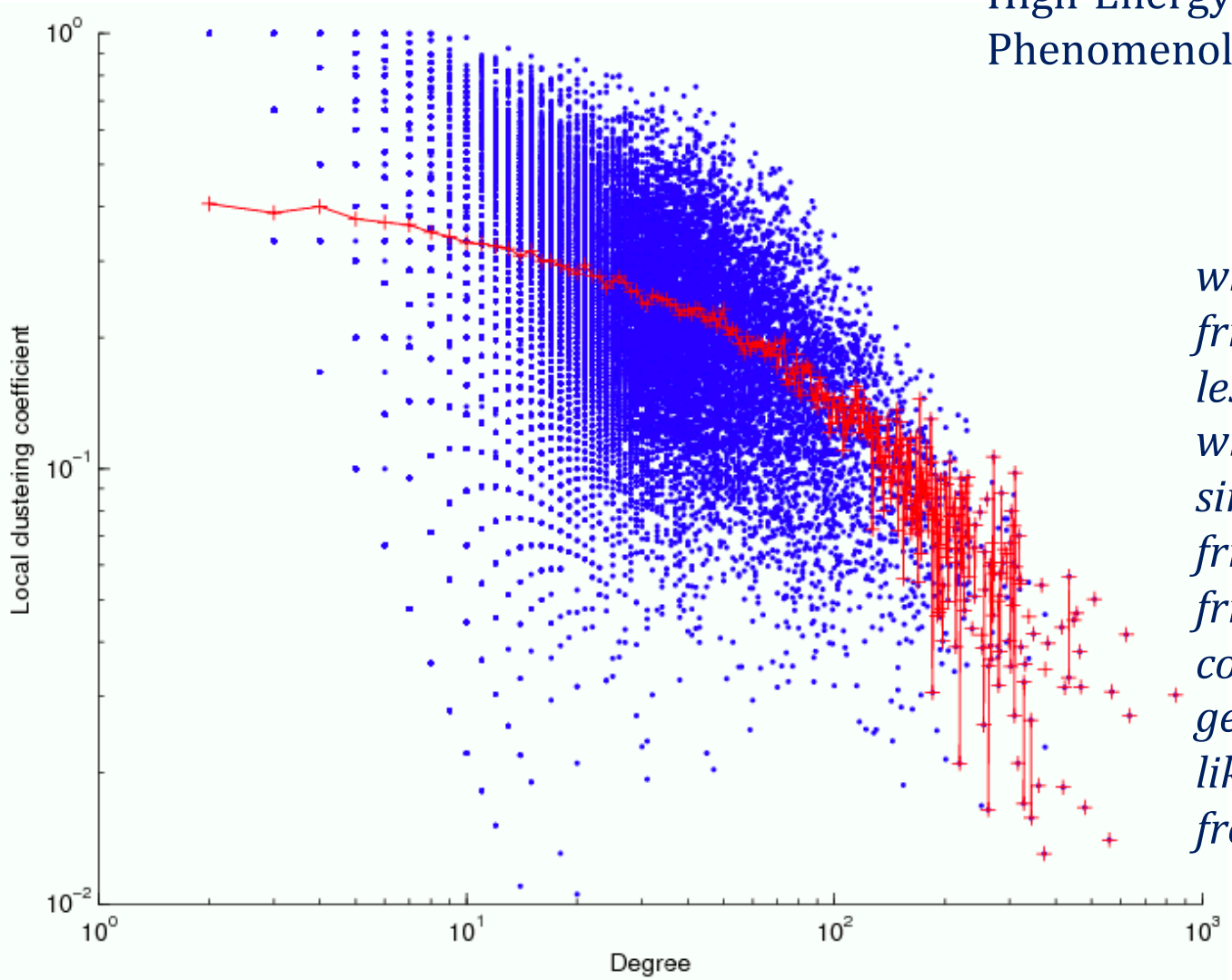
$C_1 = 1 = 6 / (4x3/2)$



$\langle C \rangle = 1$

Clustering coeff. versus degree

citation network taken from arXiv[9]s High Energy Physics / Phenomenology section



when person has many friends, these friends have less edges among them, which is to be expected since a person with many friends is likely to have friends from more diverse communities, and a paper getting cited many times is likely to be cited by papers from more diverse areas

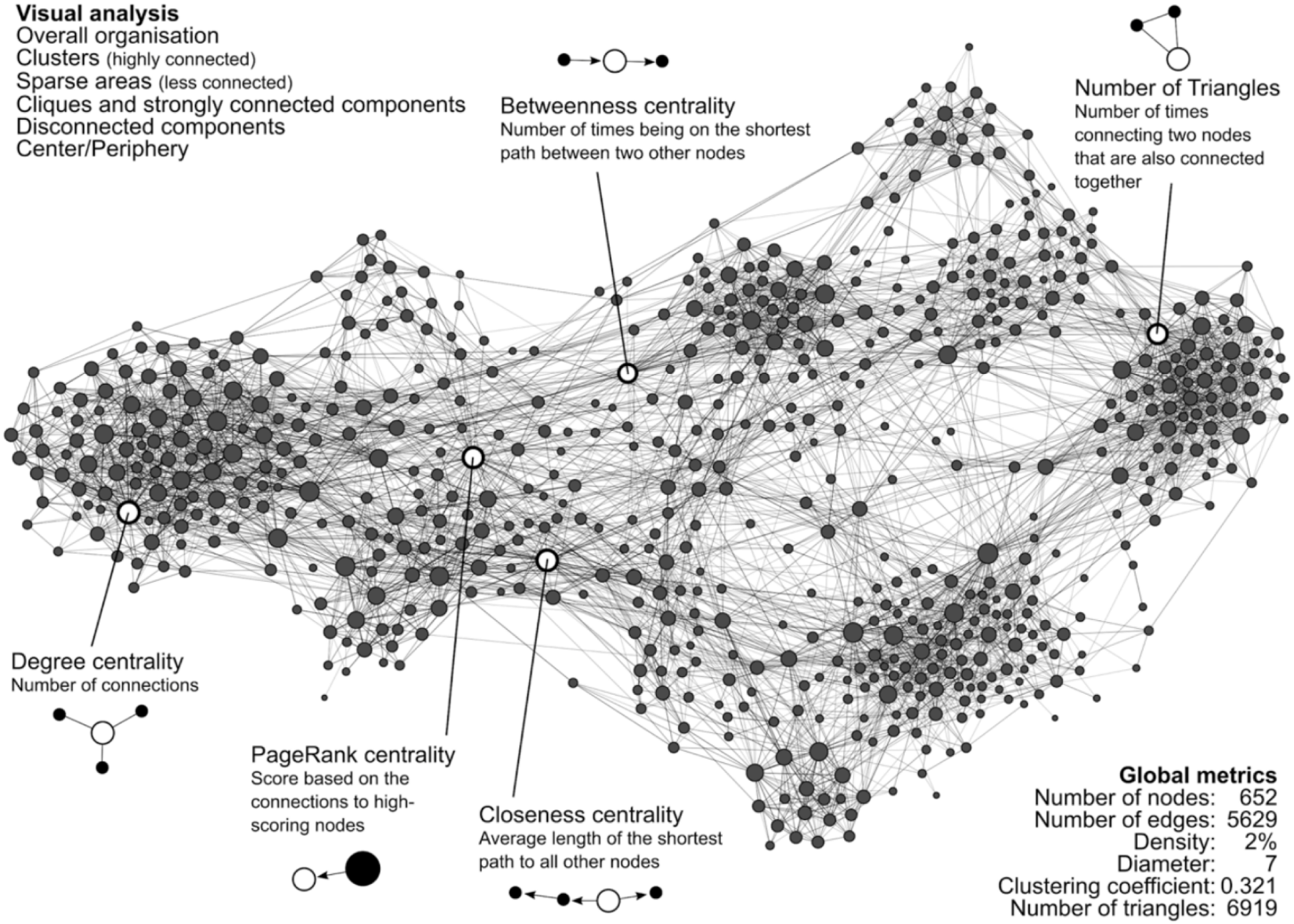
Wrap-up

Take-aways

Centrality measure	Technical property	Meaning
Degree (in/out)	Measures number (and quality) of connections	Cohesion Entrepreneurship
PageRank (authorities/hubs)	Measures number (and quality) of direct and indirect connections	Cohesion Entrepreneurship Closeness/Similarity/Friendship (with a direction) Dependence
Closeness	Measures length of min paths	Visual centrality Significant spreading points Outliers
Betweenness	Measures number of min paths	Brokerage Structural holes Ostracism
Clustering coeff.	Measures number of triadic closures	Centrality in a community Cohesion of the neighbourhood

More on the meaning…

https://reticular.hypotheses.org/1745



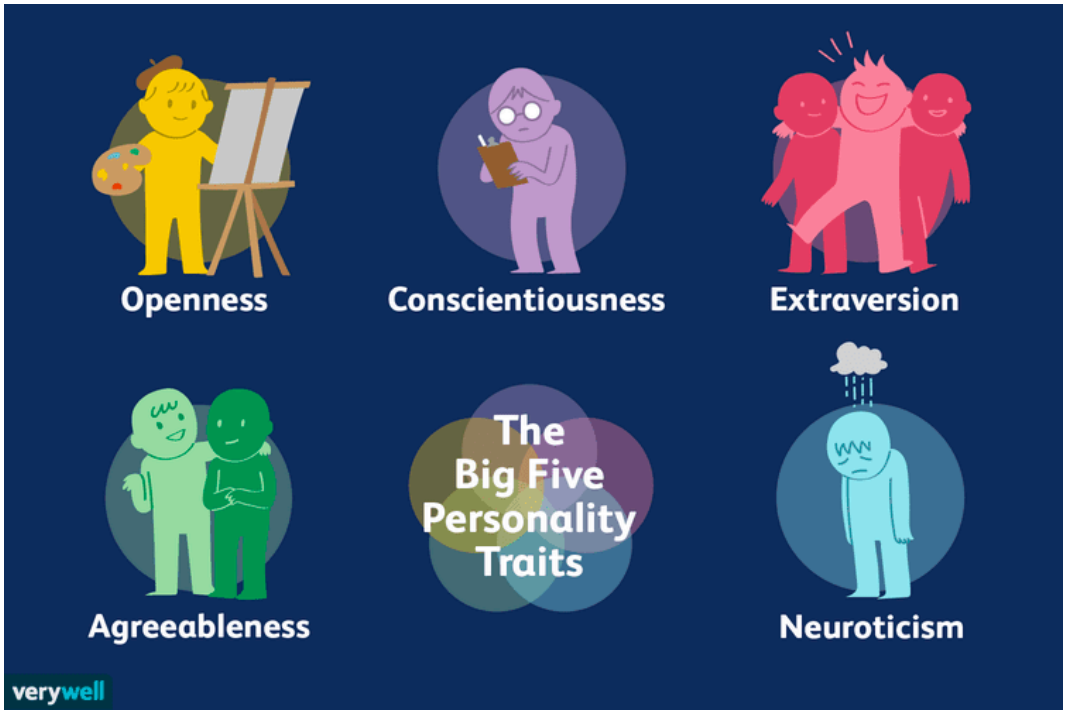
How does personality relate to centrality?

Fang, Landis, Zhang, Anderson, Shaw, Kilduff (2015).
Integrating personality and social networks: A meta-analysis of personality, network position, and work outcomes in organizations.
Organization Science, *26*(4), 1243-1260.
https://pubsonline.informs.org/doi/full/10.1287/orsc.2015.0972

- personality: big5 + self-monitoring
- network: in-degree + betweenness centrality
- performance: job performance+ career success

The big5 model

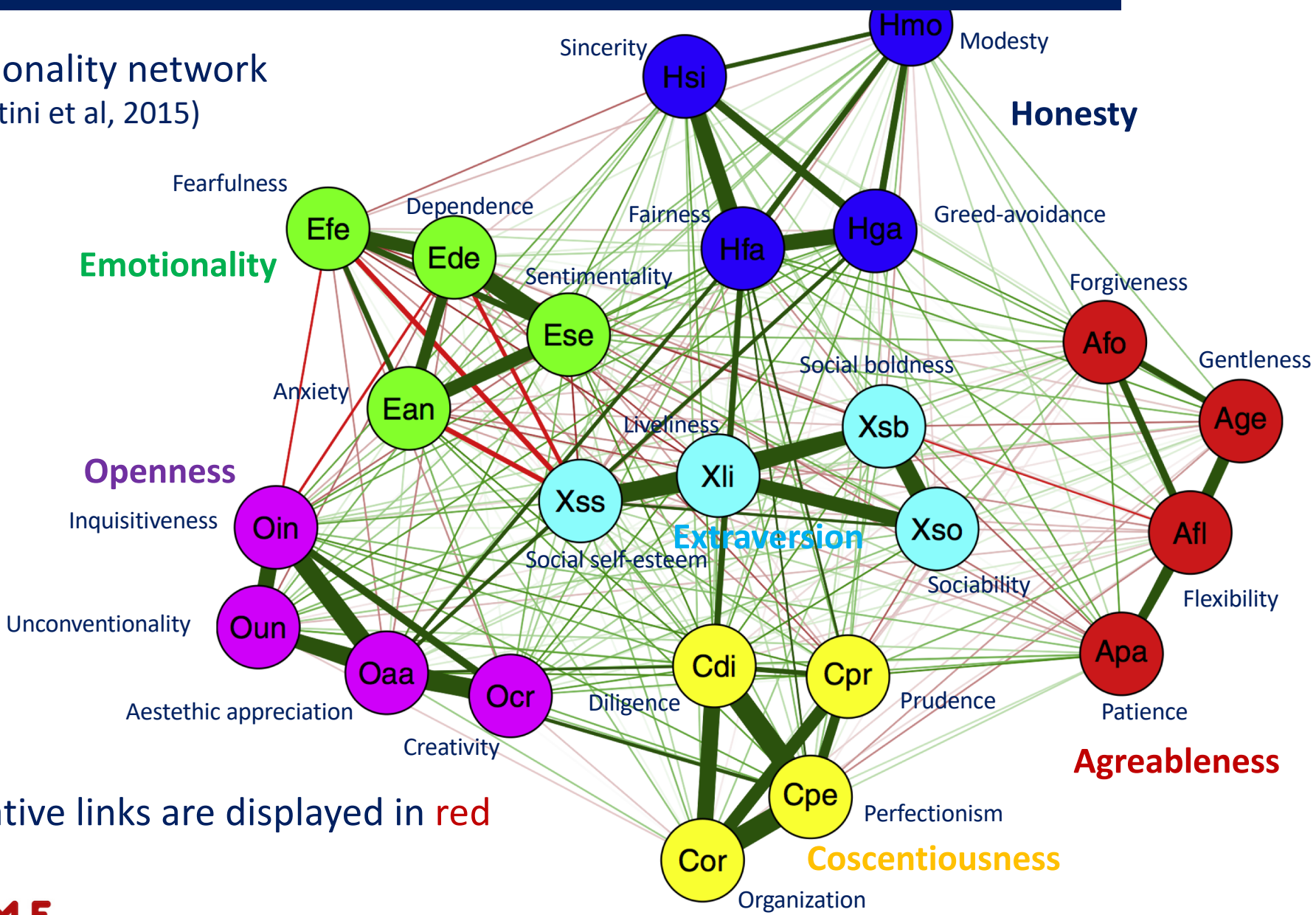
- Personality traits / dimensions
- An integration of personality research that represents the various personality descriptions in one common framework



- Individual differences in social and emotional life organized into a five-factor model of personality
- “broad abstract level and each dimension summarized a larger number of … personality characteristics” (Oliver & Srivastava, 1999)

The Big5 model relations

A personality network
(Costantini et al, 2015)



Negative links are displayed in red

MiME

Self-monitoring

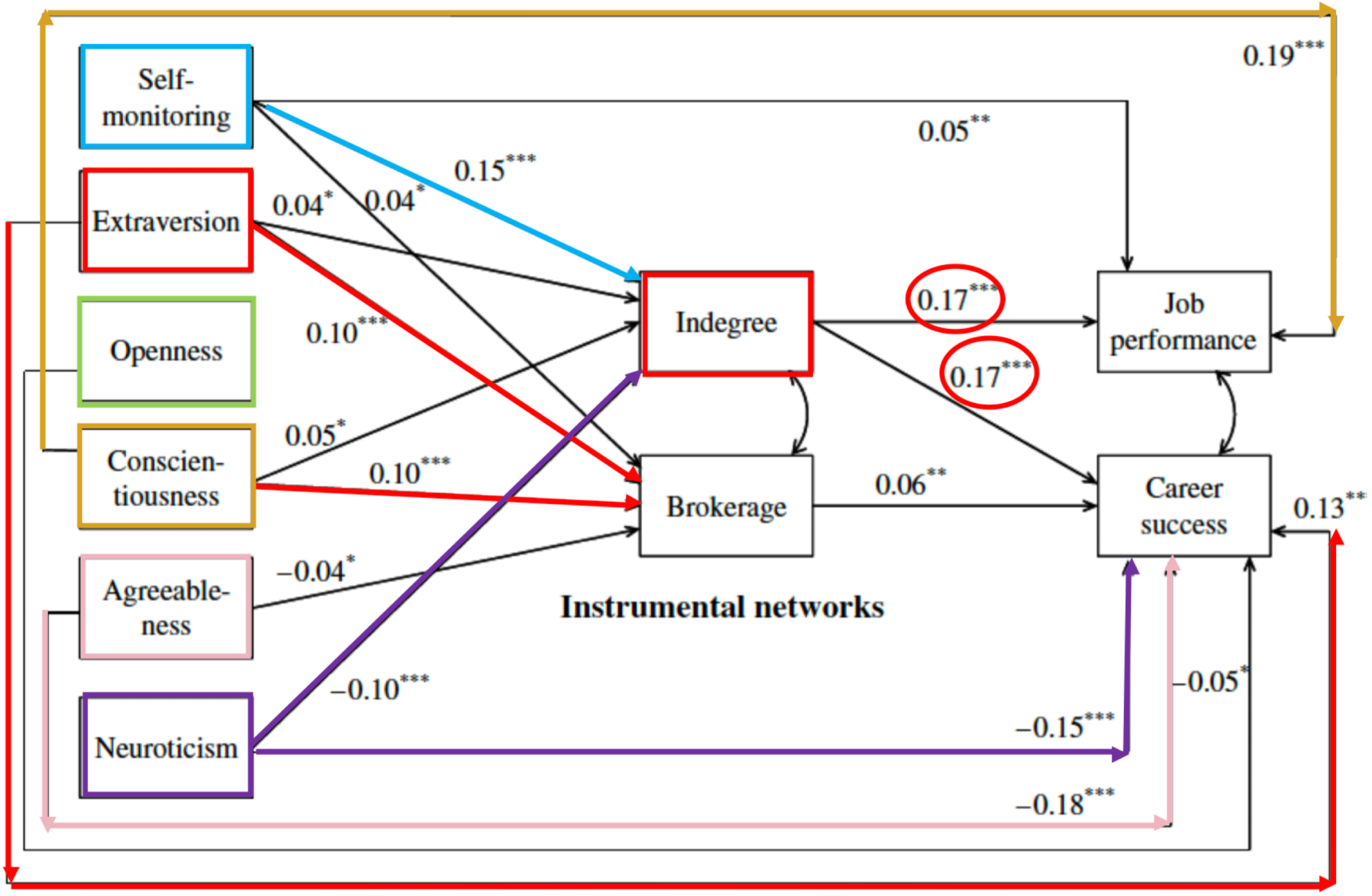
Self-monitoring

From Wikipedia, the free encyclopedia

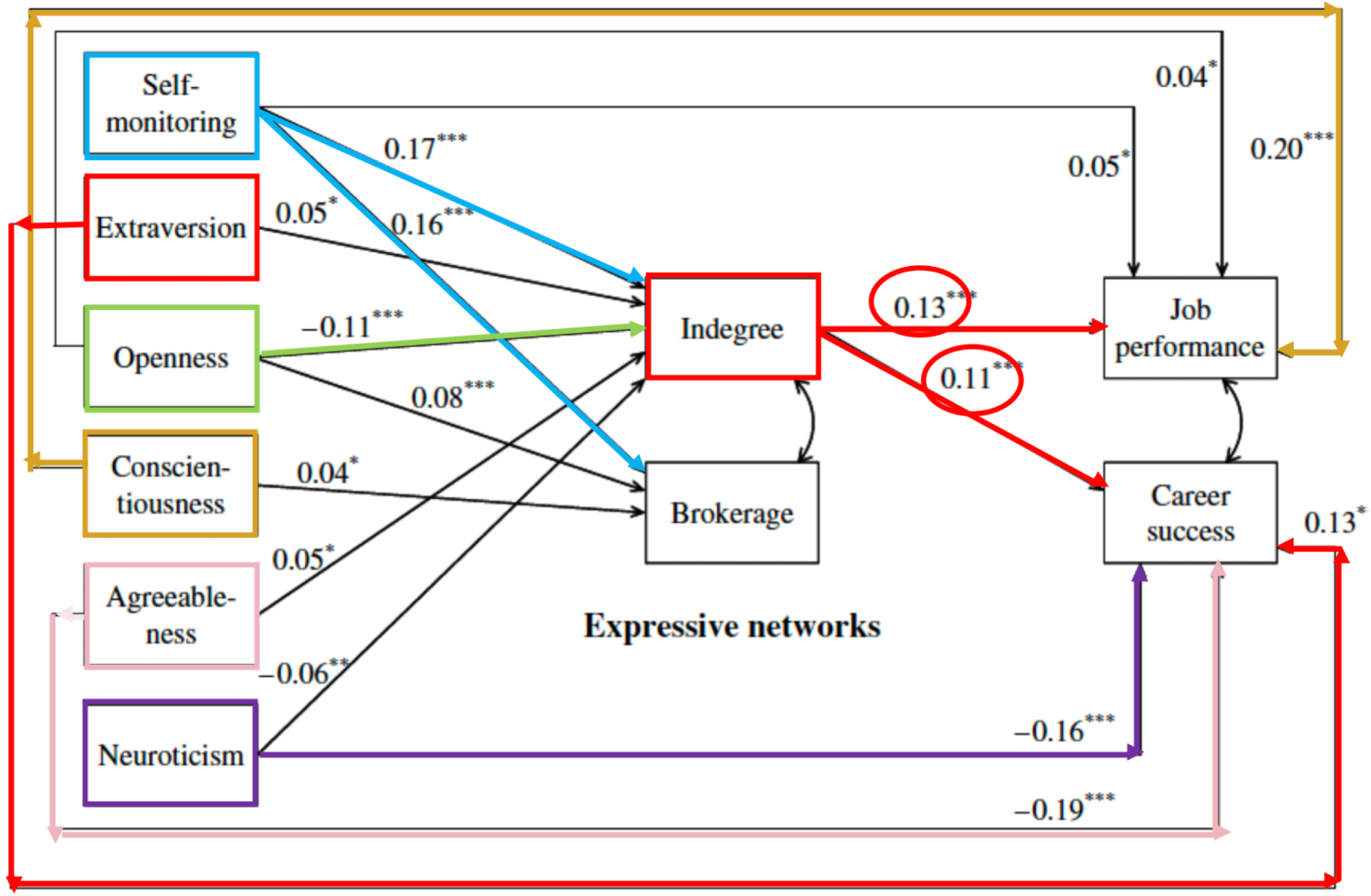
This article is about the theory. For recording of one's own activities, see Quantified Self.

Self-monitoring is a concept introduced during the 1970s by Mark Snyder, that shows how much people monitor their self-presentations, expressive behavior, and nonverbal affective displays.[1] Human beings generally differ in substantial ways in their abilities and desires to engage in expressive controls (see dramaturgy).[2] It is defined as a personality trait that refers to an ability to regulate behavior to accommodate social situations. People concerned with their expressive self-presentation (see impression management) tend to closely monitor their audience in order to ensure appropriate or desired public appearances.[3] Self-monitors try to understand how individuals and groups will perceive their actions. Some personality types commonly act spontaneously (low self-monitors) and others are more apt to purposely control and consciously adjust their behavior (high self-monitors).[4] Recent studies suggest that a distinction should be made between acquisitive and protective self-monitoring due to their different interactions with metatraits.[5] This differentiates the motive behind self-monitoring behaviours: for the purpose of acquiring appraisal from others (acquisitive) or protecting oneself from social disapproval (protective).

Instrumental network (job contacts)



Expressive network (friends)



Questions ?